International Conference on
DYNAMICAL METHODS
FOR DIFFERENTIAL EQUATIONS
in honour of George R. Sell's 65th birthday
September 4 - 7, 2002

Castillo de la Mota
Medina del Campo, Valladolid, Spain
http://wmatem.eis.uva.es/~dmde02/
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DYNAMICAL METHODS FOR DIFFERENTIAL EQUATIONS
Medina del Campo, Valladolid, Spain – September 4-7, 2002

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CONFERENCE PROGRAM

<table>
<thead>
<tr>
<th>Time</th>
<th>Wednesday 4</th>
<th>Thursday 5</th>
<th>Friday 6</th>
<th>Saturday 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>9:00 – 10:00</td>
<td>Opening</td>
<td>U. Krichgraber</td>
<td>W. de Melo</td>
<td></td>
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<tr>
<td>10:00 – 11:00</td>
<td>G. R. Sell</td>
<td>À. Jorba</td>
<td>L. Díaz</td>
<td>P. Kloeden</td>
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<tr>
<td>11:00 – 11:30</td>
<td>Coffee break</td>
<td>Coffee break</td>
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<td>Coffee break</td>
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<tr>
<td>11:30 – 12:30</td>
<td>Y. Latuskin</td>
<td>R. Krikorian</td>
<td>G. Keller</td>
<td>J. A. Rodríguez</td>
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<tr>
<td>12:30 – 13:00</td>
<td>F. Colonius</td>
<td>Sessions A3,B3,C3</td>
<td>Sessions A4,B4,C4</td>
<td>Sessions A6,B6,C6</td>
</tr>
<tr>
<td>13:30 – 15:30</td>
<td>Lunch</td>
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<tr>
<td>15:30 – 16:30</td>
<td>Y. Yi</td>
<td>R. Markarian</td>
<td>V. Baladi</td>
<td>Sessions A7,B7,C7</td>
</tr>
<tr>
<td>16:30 – 17:00</td>
<td>Coffee break</td>
<td>Coffee break</td>
<td>Coffee break</td>
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<tr>
<td>17:00 – 18:00</td>
<td>Sessions A1,B1,C1</td>
<td></td>
<td>W. Ruess</td>
<td>P. Glendinning</td>
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<tr>
<td>18:00 – 18:15</td>
<td>Break</td>
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SOCIAL PROGRAM

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<th>Day</th>
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<th>Event</th>
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<tr>
<td>Tuesday 3</td>
<td>21:00–22:30</td>
<td>Welcome Cocktail</td>
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<td>Vino español (cold buffet) at Casa de la Cultura, Medina</td>
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<tr>
<td>Thursday 5</td>
<td>17:00–23:00</td>
<td>Visit to Valladolid</td>
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<tr>
<td></td>
<td></td>
<td>Vino español (cold buffet) at Palacio de Santa Cruz, Valladolid</td>
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<tr>
<td>Friday 6</td>
<td>21:00–23:30</td>
<td>Conference Banquet</td>
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<td>Hotel San Roque, Medina</td>
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</tbody>
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LIST OF CONFERENCES

Viviane BALADI
Ruelle-Lefschetz Zeta Functions in Higher Dimension. (Kneading Determinants and Spectral Interpretation.)

Fritz COLONIUS
Some Topological Properties of Control Systems.

Lorenzo DÍAZ
Tame and Wild C¹-Generic Dynamics.

Paul GLENDINNING

Èngel JORBA
Numerical Methods for Reducibility Problems.

Gerhard KELLER
Spectral Perturbation Theory for Hyperbolic Dynamical Systems.

Urs KIRCHGRABER
On some Computer Methods in Dynamics.

Peter E. KLOEDEN
Weak Pullback Attractors of Setvalued Nonautonomous Dynamical Systems.

Raphael KRIKORIAN
Reducibility, Differentiable Rigidity and Positive Lyapunov Exponents for Quasi-Periodic Cocycles: Non Perturbative Results.

Yuri LATUSHKIN
Evolution Semigroups, Lyapunov-Oseledets Exponents, and Linearized Euler Equations.

Rafael DE LA LLAVE

Roberto MARKARIAN
Bernoulli Elliptical Stadia.

Welington de MELO
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J. Ángel RODRÍGUEZ
A View to the Genesis of Non-Hyperbolic Strange Attractors.

Wolfgang RUESS

George R. SELL
The Search for El Niño: An Application of Quasi-Periodic Dynamics.

Yingfei YI
On Almost Automorphic Dynamics.
LIST OF SESSIONS

Session 1 – WEDNESDAY 4 – 17:00 – 18:00

A1 – Conference Room A
17:00–17:15 M. Nerurkar On a Conjecture of A. Forrest.

B1 – Conference Room B
17:00–17:15 J. Galán Stability and Bifurcation Behavior of the Figure Eight Solution of the Three Body Problem.
17:40–17:55 J.L. Bravo The Number of Bifurcation Points of a Periodic ODE with Quadratic Nonlinearities.

C1 – Conference Room C
17:00–17:15 M. Gidea A Topological Model for Arnold Diffusion.

Session 2 – WEDNESDAY 4 – 18:15 – 19:15

A2 – Conference Room A
18:15–18:30 E. Ponce Limit Cycle Bifurcation from Infinity in Symmetric 3D Piecewise Linear Systems.
18:55–19:10 R. Riaza Multiple Singular Bifurcation in Matrix Pencils and DAEs.

B2 – Conference Room B
18:15–18:30 D. Cheban An Analog of the Cameron-Johnson Theorem.

C2 – Conference Room C
18:15–18:30 J. Villanueva Asymptotic Size of Herman Rings of the Complex Standard Family.
Session 3 – THURSDAY 5 – 12:30 – 13:30

A3 – Conference Room A
12:30–12:45  Ll. Alsedà  A Note on the Periodic Orbits and Topological Entropy of Graph Maps.

B3 – Conference Room B
12:30–12:45  A. Cordero  Integrable Hamiltonian Systems on S2xS1.

C3 – Conference Room C

Session 4 – FRIDAY 6 – 12:30 – 13:30

A4 – Conference Room A
12:30–12:45  L. Górniiewicz  On the Lefschetz Fixed Point Theorem.
12:50–13:05  I. Baldomá  Invariant Manifolds Associated to Degenerated Fixed Points for Maps in R^n.

B4 – Conference Room B
12:30–12:45  G. Del Magno  Bernoulli Elliptical Stadia II.

C4 – Conference Room C
12:50–13:05  A. Olek  Continuous Dependence on Obstacles in Double Global Obstacle Problems.
Session 5 – FRIDAY 6 – 18:15 – 19:15

A5 – Conference Room A

B5 – Conference Room B
18:35–18:50 P. Casas  Numerical Continuation of Unstable Quasi-Periodic Orbits for the 2-D Poiseuille Flow.

C5 – Conference Room C
18:35–18:50 C. Alonso  Topological Equivalence for Chains of Saddles.

Session 6 – SATURDAY 7 – 12:30 – 13:30

A6 – Conference Room A
12:30–12:45 T. Caraballo  Pullback Attractors for Dynamical Systems with Delays.

B6 – Conference Room B

C6 – Conference Room C
12:30–12:45 I. Djellit  Homoclinic Bifurcations in a Two-Dimensional Endomorphism.
Session 7 – SATURDAY 7 – 15:30 – 16:30

A7 – Conference Room A
15:50–16:05 M. Mazur  Shadowing and Likes as $C^0$ Generic Properties.

B7 – Conference Room B

C7 – Conference Room C
15:50–16:05 H. Moshchev  Recurrence of Solutions of Differential Equations with Almost- and Quasiperiodic Coefficients.
BALADI, Viviane.

Title: Ruelle-Lefschetz Zeta Functions in Higher Dimension. (Kneading Determinants and Spectral Interpretation.)

Abstract: We shall present work of Baillif and Baillif and myself, extending the one-dimensional results on sharp determinants (generalised weighted Lefschetz dynamical zeta functions) of Ruelle and myself [Inv. Math 1996] to higher dimensions, under a transversality assumption. The original ideas can be traced back to Milnor and Thurston, but the introduction of weights make the analysis more interesting. We are able to connect the zeroes and poles of the sharp determinant to eigenvalues in some cases.

Keywords: Dynamical zeta functions, transfer operators, spectrum, kneading determinants.

COLONIUS, Fritz.

Title: Some Topological Properties of Control Systems.

Abstract: Control systems which are a special class of nonautonomous differential equations may be viewed as topological skew product flows. Their topologically transitive components are given by the local control sets, i.e., subsets of complete controllability. Some of their topological properties will be discussed by constructing an associated fundamental semigroup, based on homotopies within the set of trajectories.

This talk is based on joint work with M. Spadini (Florence) and with E. Kizil and L. San Martin (Campinas).

DÍAZ, Lorenzo.

Title: Tame and Wild $C^1$-Generic Dynamics.

Abstract: Mañé stated a dichotomy between hyperbolicity and coexistence of infinitely many sources or sinks for $C^1$-generic diffeomorphisms of surfaces. We discuss this result in higher dimensions and study some pathological dynamics that can appear in higher dimensions (besides the persistent coexistence of infinitely many sinks or sources). We pay special attention to the creation of minimal transitive sets associated to homoclinic classes that do not admit any dominated splitting.

GLENDINNING, Paul.

Title: Pinching, Strange Nonchaotic Attractors and Generalized Pitchfork Bifurcations in Quasi-Periodically Forced Maps.

Abstract: We give some extensions of results by Keller on the existence of strange nonchaotic attractors in quasi-periodically forced systems. These results make it possible to describe a codimension two pitchfork bifurcation of involving strange nonchaotic attractors using a simple example.

JORBA, Àngel.

Title: Numerical Methods for Reducibility Problems.

KELLER, Gerhard.

Title: Spectral Perturbation Theory for Hyperbolic Dynamical Systems.

Abstract: In this talk I will report about two related results:


[http://www.mi.uni-erlangen.de/~keller/publications/bkl-paper.ps.gz]

I) is a spectral stability result that assumes only that an operator and its perturbations satisfy uniformly an Lasota-Yorke-type inequality. In one-dimensional dynamics it applies to piecewise expanding maps and their perturbations by noise, small distortions or discretization.

II) describes a functional analytic setting which allows to study the transfer operators of Anosov diffeomorphisms as quasi-compact operators and to apply the perturbation theorem from I) to smooth perturbations of Anosov diffeomorphisms. (At the present stage the perturbation results are proved for two-dimensional diffeomorphisms only.)

**KIRCHGRABER, Urs.**

**Title:** On some Computer Methods in Dynamics.

**Joint work with:** STOFFER, Daniel.

**Abstract:** In this talk the first author reviews recent work on the use of computers to establish chaotic, periodic and stable behaviour in dynamical systems, with applications to Celestial Mechanics. Various levels of rigour are briefly discussed.

**KLOEDEN, Peter E.**

**Title:** Weak Pullback Attractors of Setvalued Nonautonomous Dynamical Systems.

**Abstract:** Weak pullback attractors are defined for setvalued nonautonomous dynamical systems and their existence and upper semicontinuous convergence under perturbation is established for both the discrete and continuous time cases. Unlike the usual strong pullback attractors, invariance and pullback attraction here are required only for (at least) a single trajectory rather than all trajectories at each starting point. The concept is thus useful, in particular, for discrete time control systems.

**KRIKORIAN, Raphael.**

**Title:** Reducibility, Differentiable Rigidity and Positive Lyapunov Exponents for Quasi-Periodic Cocycles: Non Perturbative Results.

**Abstract:** We prove a non-perturbative reducibility theorem for smooth quasi-periodic (diophantine rotation number in the base) cocycles with values in $SL(2,\mathbb{R})$ which has two corollaries. The first one is a differentiable rigidity theorem: if such a cocycle is continuously reducible (that is conjugated to a generic constant system via a continuous conjugacy) then the conjugacy is smooth.

The second one is: if the rotation on the base is fixed and diophantine then the set of cocycles for which the (maximal) Lyapunov exponent is positive (non zero) is dense in the smooth topology.

**LATUSHKIN, Yuri.**

**Title:** Evolution Semigroups, Lyapunov-Oseledets Exponents, and Linearized Euler Equations.

**Abstract:** We study spectral properties of the linearized Euler operator for an ideal incompressible fluid in dimensions two and three. The main tool in our analysis is the techniques of constructing approximate eigenfunctions that were recently developed for so-called evolution semigroups. The evolution semigroup is a semigroup of transfer-type operators induced by a given flow.

We give information about the essential spectrum of the linearized Euler operator and describe the linearized hydrodynamic stability in terms of the spectrum of the linearized operator thus proving a spectral mapping theorem for the corresponding group. In particular, the boundaries of the essential spectrum are described in terms of Lyapunov-Oseledets exponents given by the Multiplicative Ergodic Theorem. Also, we relate the spectrum of the linearized Euler operator and Lyapunov exponents of the bicharacteristic amplitude system, a system of ordinary differential equations whose asymptotic behavior is responsible for the description of the essential spectrum of the linearized Euler operator.
DE LA LLAVE, Rafael.

Title: Geometric Mechanisms for Topological Instability in Hamiltonian Systems.

Joint work with: DELSHAMS, Amadeu and SEARA, Teresa.

Abstract: It is well known (and probably true) that typical mechanical systems with more than 3 degrees of freedom are topologically instable (a typical trajectory will explore regions of phase space that get close enough to wide regions of phase space).

Such result is clearly false for integrable systems. Perturbations of integrable systems present obstructions to diffusion such as KAM tori and Nekhoroshev estimates.

A well known example of Arnol’d ’64 showed that, in spite of these obstructions, topological instability is possible in near integrable systems.

This remarkable example relies on the fact that perturbation enjoyed several non-generic properties. Notably, an absence of resonances.

In contrast, numerical experimentation and heuristic discussion (Chirikov, Tennyson, Lieberman, etc.) suggests that it is resonances which cause the strongest instability.

We will present a mathematically rigorous treatment of several geometric mechanisms of instability in near integrable systems which take place near resonances.

MARKARIAN, Roberto.

Title: Bernoulli Elliptical Stadia.

Joint work with: DEL MAGNO, Gianluigi.

Abstract: Let $Q_{a,h}$ be a convex region of the plane whose boundary consists of two semiellipses joint by two (straight) lines parallel to the major axis of the semiellipses (elliptical stadium). The axis of the semiellipses have length 2 and $2a$, $a > 1$, and the lines have length $2h$. For $1 < a < \sqrt{4 - 2\sqrt{2}}$ and $h > 2a^2\sqrt{a^2 - 1}$, we give a complete proof of the following result: the billiard map in the elliptical stadium $Q_{a,h}$ is ergodic, K-mixing and Bernoulli with respect to the natural billiard measure.

This talk will focus on the general structure of the proof and its applicability to a general class of billiard maps. The complementary talk of Del Magno will explain some of its most important steps.

DE MELO, Welington.

Title: Renormalization in Dynamics, Statistical Mechanics and Quantum Field Theory.

Abstract: I will survey some recent progress that led to the complete understanding of the dynamics of the renormalization operator in one-dimensional dynamics and I will discuss some related concepts and problems in Statistical Mechanics and Quantum Field Theory.

RODRÍGUEZ, José Ángel.

Title: A View to the Genesis of Non-Hyperbolic Strange Attractors.

Abstract: The existence of strange attractors seems to be the most interesting asymptotic behaviour in dissipative dynamical systems. Hyperbolic strange attractors were early constructed (Smale, 1967). These attractors are completely persistent and structurally stable, therefore the unstable and (or) non-persistent ones that had been found in numerical experiments (Lorenz attractor, for instance) could not be hyperbolic. Nevertheless, these attractors look like strange ones but, does there really exist non-hyperbolic strange attractors?

This talk deals with this question. From the first proof of the existence and persistence of non-hyperbolic strange attractors given by Benedicks and Carleson in 1991, we shall go to three-dimensional scenery where a one-parameter family of vector fields have simultaneously infinitely many persistent non-hyperbolic strange attractors (the lack of genericity keeps still open the conjecture of J. Palis). We shall also explain how this scenery can be generically unfolded from a nilpotent singularity of codimension four. Whether this is or not the lowest possible codimension is still an open problem.
RUSS, Wolfgang M.

Title: Existence and Flow Invariance for Solutions to Nonlinear Partial Differential Delay Equations.

Abstract: The object of the talk is a study of partial differential delay equations of the form

\[
(FDE) \quad \begin{cases}
\dot{x}(t) + Bx(t) \ni F(x_I) , \quad t \geq 0 \\
x|_I = \varphi \in \hat{E}.
\end{cases}
\]

where $B \subset X \times X$ is a (generally) nonlinear and multivalued differential expression in a Banach space $X$, and for given $I = [-r, 0]$, $r > 0$ (finite delay), or $I = \mathbb{R}^-$ (infinite delay), and $t \geq 0$, $x_I : I \to X$ is the history of $x$ up to $t : x_I(s) = x(t + s)$, $s \in I$. Moreover, $\varphi : I \to X$ is a given initial history out of a space $E$ of functions from $I$ to $X$, and $F$ is a given history-responsive operator with domain $\hat{E} \subset E$ and range in $X$.

We study the existence of mild solutions to (FDE) and their asymptotic behaviour. In particular, we present a condition for the flow-invariance of specific subsets $\hat{E}$ of the initial history space $E$, that extends the known sufficient conditions for the special cases of (1) $B = 0$, or (2) $B$ linear and $-B$ generating a $C_0$-semigroup, or (3) $B$ m-accretive and $I = \{0\}$ (no delay) by Bothe, Leela/Moauro, Lightbourne, Martin, Martin/Smith, Pazy, Seifert, Vrabie and others to the general nonlinear and delay case of the above problem (FDE).

The results apply to diffusive population models with temporal or spatio-temporal averages over the past history, such as

\[
\begin{cases}
\dot{u}(t) - \Delta u(t) \ni au(t) \left[1 - bu(t) - \int_{-\infty}^{t} \int_{\Omega} g(\cdot - y, t - s)u(s)(y)dyds\right] , \quad t \geq 0 \\
u|_{(-\infty,0]} = \varphi
\end{cases}
\]

as well as corresponding models with the Laplacian being replaced by more general and nonlinear diffusion/absorption operators of the form

\[-div a(\cdot, \text{grad } u) + \beta(u); \quad -a(\cdot, \text{grad } u) \cdot n \in \beta(u) \quad \text{on } \partial\Omega\]

with $\beta \subset \mathbb{R} \times \mathbb{R}$ a maximal monotone graph.

SELL, George R.

Title: The Search for El Niño: An Application of Quasi-Periodic Dynamics.

YI, Yingfei.

Title: On Almost Automorphic Dynamics.

Abstract: Almost automorphy is a notion first introduced by S. Bochner in 1955 to generalize the almost periodic one. It is proven to be a fundamental notion in characterizing multi-frequency phenomena and their generating dynamical complexity. This lecture will give a survey of the topic along with some discussions on related problems arising in almost periodically forced differential equations, non-linear oscillators, and lattice dynamical systems.
Ait Dads, E.

**Title:** Existence and Almost Periodicity for Some State-Dependent Delay Differential Equations.

**Joint work with:** Ezzinbi, E.

**Abstract:** In this work we study the existence and uniqueness of solutions for some state dependent delay differential equation of type

\[
\begin{align*}
\frac{d}{dt} x(t) &= F(t, x(t), x(t - \rho(x_t)), \text{ for } t \geq 0 \\
x_0 &= \varphi \in C = C([0, 0], \mathbb{R}^n)
\end{align*}
\]

When the nonlinear term is almost periodic, we prove the existence of an almost periodic solution. According to the book of Hale, it’s well known if the term \( F \) is continuous, the equation has at least one maximal solution \( x(\cdot, \varphi) \) which is defined on some interval \( [0, t_\varphi] \) and \( t_\varphi \) is infinite or finite and in this case we have the blow up of solution:

\[
\lim_{t \to t_\varphi} |x(t, \varphi)| = \infty.
\]

The uniqueness is not true, even if \( F \) is lipschitzian with respect to the second argument, the uniqueness can be proved only for lipschitzian initial data \( \varphi \), for this topics we refer to the work of Mallet-Parret. W. Alt proved the existence and periodicity for some state dependent delay differential equation. Recently Louihi has proved the existence and uniqueness of solution for some autonomous state dependent delay differential equations, the authors have proved that the solutions generate a strongly semigroup on the space of Lipschitz continuous function \( \varphi \). Arino, Hadeler and Hbid have proved also the existence of oscillatory and periodic solutions for some state dependent delay arising from population dynamic problems.

Alonso, C.

**Title:** Topological Equivalence for Chains of Saddles.

**Abstract:** We study the topological equivalence between two analytic vector fields in \( \mathbb{R}^3 \) defined on a neighbourhood of an invariant, closed and simple curve \( E \). We will suppose that the singular points are hyperbolic and the curve \( E \) is the intersection of two invariant and transversal surfaces.

When sinks or sources appear, we reduce the problem by splitting the curve \( E \). The main difficulty is the study of the topological equivalence for linear or circular chains of saddle connections. The knowledge of the invariants for the topological equivalence between two saddle connections will allow us to develop the compatibility properties to extend the equivalence to the lineal or circular chains.

The key technic tool will be the monomial blowing-up with different weights and center the curve \( E \) and also the compatible foliations with the saddles in transversal discs to \( E \).

Alsedà, Ll.

**Title:** A Note on the Periodic Orbits and Topological Entropy of Graph Maps.

**Joint work with:** Juher, D. and Mumbrú, P.

**Abstract:** This paper deals on the relationship between the periodic orbits of continuous maps on graphs and the topological entropy of the map. We show that the topological entropy of a graph map can be approximated by the entropy of its periodic orbits.


**Keywords:** Graph maps, periodic orbits, topological entropy.
ALVES, J.

Title: Markov Structures and Rates of Mixing for Non-Uniformly Expanding Dynamical Systems.

Abstract: We consider non-uniformly expanding maps on compact Riemannian manifolds of arbitrary dimension, possibly having discontinuities and/or critical sets, and show that under some general conditions they admit an induced Markov tower structure for which the decay of the return time function can be controlled in terms of the time generic points need to achieve some uniform expanding behavior. As a consequence we obtain some rates for the decay of correlations of those maps and conditions for the validity of the Central Limit Theorem.

ARAUJO, V.

Title: On the Uniform Hyperbolicity of Some Nonuniformly Hyperbolic Systems.

Abstract: In a joint work with J. F. Alves (Univ. Porto, Portugal) and B. Saussol (Univ. Picardie, France), we give sufficient conditions for the uniform hyperbolicity of certain nonuniformly hyperbolic dynamical systems. In particular, we show that local diffeomorphisms that are nonuniformly expanding on sets of total probability (probability one with respect to every invariant probability measure) are necessarily uniformly expanding. We also present a version of this result for diffeomorphisms with nonuniformly hyperbolic sets.

AVILA, A.

Title: Statistical Properties of (Smooth) Unimodal Maps.

Joint work with: MOREIRA, C.G.

Abstract: We show that typical smooth unimodal maps have a good statistical description: they are either hyperbolic or have a renormalization supporting an absolutely continuous invariant measure which is exponentially mixing and stochastically stable. This proves the Palis conjecture in the unimodal case. Here we consider Kolmogorov’s notion of typical: our description is valid for a full measure set of parameters in some (topologically) generic set of smooth families. The proof uses the work of Lyubich, de Melo and myself on the structure of spaces of analytic unimodal maps to extend the results of Moreira and myself on the quadratic family.

BALDOMÁ, I.

Title: Invariant Manifolds Associated to Degenerated Fixed Points for Maps in $\mathbb{R}^n$.

Joint work with: FONTICH, E.

Abstract: The invariant manifolds associated to invariant objects of a dynamical system, give us an essential information in order to analyze the dynamical structure of the dynamical system. If the invariant object has some hyperbolicity, there are satisfactory results about the existence, regularity and uniqueness of the invariant manifolds in high dimensions. See for instance [1], [3].

But, when the invariant object that we are considering has not got any hyperbolic “direction” the problem is more intricated. In dynamical systems generated by maps, the non hyperbolicity condition of a fixed point, is just that the eigenvalues of the differential of the map evaluated to the fixed point have got module 1. The case that all the eigenvalues are exactly 1 is the most degenerate. For planar maps, this case have been considered by [2], [4].

We consider maps of the form $(x, y) \to (x + p(x, y), y + q(x, y))$ where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$ and $p, q$ are $o(||(x, y)||^2)$.

We have proved that, under suitable hypotheses, there exists invariant manifold associated to the origin and that it can be written as the graph of a function $\varphi$. We have considered two cases, when the map is a Lipschitz function and when it is analytic and we have proved that $\varphi$ is also Lipschitz and analytic, respectively.

Our hypotheses are a generalization to higher dimensions of the ones in [4]. Roughly speaking these hypotheses generate a weak hyperbolicity.
The maps that we have studied, appear in some problems of celestial mechanical and we believe that our result could apply in problems of fluids mechanics.

References:

BERNARD, P.

Title: The Minimal Action Near Positive Definite Invariant Tori.

Abstract: Near a KAM torus of a symplectic map, two objects can be defined. The Birkhoff invariants on one hand, the averaged action introduced by Mather on the other hand. We show that these two objects are closely connected. This was first noticed by Siburg in the neighborhood of elliptic fixed point. The extension to higher dimension of this remark necessitates some modifications in the proof, which incidentally makes it much clearer. As a corollary we obtain that the Birkhoff invariants depend only on the action spectrum of periodic orbits near the invariant torus.

Keywords: Averaged action, minimizing orbits, Aubry Mather, KAM tori, Birkhoff normal form.

BERTI, M.

Title: Drift in Phase Space: a New Variational Mechanism with Optimal Diffusion Time.

Abstract: We consider non-isochronous a priori unstable Hamiltonian systems with a \(O(\mu)\) perturbation which does not preserve the unperturbed tori. We prove the existence of Arnold diffusion with diffusion time \((1/\mu) \ln(1/\mu)\) by a variational method which does not require the existence of “transitions chains of tori” provided by KAM theory. We also prove that our estimate of the diffusion time is optimal as a consequence of a general stability result proved via classical perturbation theory.

BRAVO, J.L.

Title: The Number of Bifurcation Points of a Periodic ODE with Quadratic Nonlinearities.

Joint work with: TINEO, A.

Abstract: We study the number of bifurcation values of the parametric periodic ODE

\[ x' = f(t, x) + \lambda x \]

when the number of periodic solutions of that equation is bounded by two for every \(\lambda\), and \(f(t, x)\) is coercive. We divide our study in two cases, when \(f(t, 0)\) has definite sign and when \(f(t, 0)\) has not definite sign.

Keywords: Periodic solutions, bifurcation, number of solutions.

CARABALLO, T.

Title: Pullback Attractors for Dynamical Systems with Delays.

Abstract: The theory of global attractors for autonomous systems as developed by Hale in [1] owes much to examples arising in the study of retarded functional differential equations. Although the classical theory can be extended in a relatively straightforward manner to deal with time-periodic equations, general non-autonomous equations such as

\[ x'(t) = F(t; x(t), x(t - r(t))) \]

(1)
fall outside its scope.

Recently, a theory of ‘pullback attractors’ has been developed (see Kloeden & Schmalfuss [2]) which allows many of the ideas for the autonomous theory to be extended to deal with such examples. However, until now this has only been applied to ordinary and partial differential equations.

It is our intention here to show how pullback attractors can be used to investigate the behaviour of non-autonomous delay equations. In particular, we are able to compare the dynamics of systems of ordinary differential equations with that of the same system with a small delay, and show that these are ‘close’ in some global sense.

**Keywords:** Differential systems with delays, pullback attractor, pullback absorption.

**References:**

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**CARBALLO, C.M.**

**Title:** $C^1$ Generic Properties of Homoclinic Classes.

**Joint work with:** Morales, C.A. and Pacífico, M.J.

**Abstract:** A homoclinic class of a vector field is the closure of the set of transverse homoclinic points associated to a hyperbolic periodic orbit.

We prove the following properties.

1. The homoclinic classes of generic $C^1$ vector fields on $n$-manifolds are maximal transitive sets, they are saturated sets and isolated if and only if $\Omega$-isolated.
2. Generic $C^1$ vector fields do not exhibit cycles associated to homoclinic classes.
3. Codimension 1 singularities, i.e. with a unique positive or negative eigenvalue, of generic $C^1$ vector fields are contained in maximal transitive sets.
4. Generic $C^1$ vector fields with finitely many homoclinic classes have finitely many attractors the union of the basins of which form an open dense set of the manifold.
5. There are locally residual sets of $C^1$ vector fields on a 5-manifold exhibiting finitely many attractors and repellers but infinitely many homoclinic classes.

We also show a sufficient condition for an attracting set to be $C^1$ weakly robust. Let us observe that these results generalize well known properties of Axiom A vector fields.

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**CASAS, P.S.**

**Title:** Numerical Continuation of Unstable Quasi-Periodic Orbits for the 2-D Poiseuille Flow.

**Joint work with:** Jorba, À.

**Abstract:** The Poiseuille problem is described as the flow of a viscous incompressible fluid in a channel between two parallel walls. As boundary conditions we suppose no-slip on the channel walls and, at artificial boundaries in the stream direction $x$, a period $L = 2\pi/\alpha$, being $\alpha$ the parameter wavenumber. The fluid is driven through an imposed constant pressure gradient. By using the numerical discretization of the Navier-Stokes equations considered in [1], we obtain different configurations of the flow, such as periodic and quasi-periodic solutions on time. Periodic flows are born at a Hopf bifurcation of the laminar solution (the basic stationary flow), as the Reynolds number, $Re$, crosses a critical value. The curve of periodic flows as a function of $Re$, also presents Hopf bifurcations, which give rise to quasi-periodic orbits.

This work is focused on the numerical continuation of quasi-periodic flows which emanate from the first Hopf bifurcation, say at $Re_1$, of periodic flows. In [2], those solutions are obtained sub-critically, i.e. they are born for $Re < Re_1$ and besides they are stable to small perturbations with equal $\alpha$. By improving the precision employed in [2], we find that the bifurcating flows are unstable and go forward with respect to $Re$. We obtain several curves of unstable quasi-periodic solutions for different values of $\alpha$. The proposed numerical scheme is based on a full numerical integration of
the Navier-Stokes equations and the use of suitable Poincaré sections. The most intensive part of
the computations has been done in parallel. We believe that this methodology can also be applied
to similar problems.

Finally the dynamics is analyzed when, instead of the pressure gradient, the volume flux per unit
span in the channel is held constant.

**Keywords:** Navier-Stokes equations, unstable flows, Hopf bifurcation.

**References:**
[1] Casas, P.S., Jorba, À., Hopf bifurcations to quasi-periodic solutions for the two-dimensional
[http://www-ma1.upc.es/~casas/trabajos/cuasiper-art.ps.gz]
[2] Soibelman, I., Meiron, D.I., Finite-amplitude bifurcations in plane Poiseuille flow: two-

**CHEBAN, D.**

**Title:** An Analog of the Cameron-Johnson Theorem.
**Abstract:** The well-known theorem asserts that the equation

\[
x' = A(\omega t)x \quad (\omega \in \Omega)
\]

(where \((\Omega, \mathbb{R}, \sigma)\) is a compact minimal dynamical system and \(\omega t := \sigma(t, \omega)\)) can be reduced by a
Lyapunov-Perron transformation to the equation

\[
x' = B(\omega t)x \quad (\omega \in \Omega)
\]

with a skew-symmetric matrix \(B(\omega)\), if all solutions of all equations (1) are bounded on the whole
line. In the talk, a generalization of this result to linear equation (1) with nonminimal \((\Omega, \mathbb{R}, \sigma)\) (\(\Omega\)
is compact and every point \(\omega \in \Omega\) is Poisson’s stable) and for the almost periodic equations in a
Banach space is presented.

**Keywords:** Cameron-Johnson theorem, Poisson’s stability, linear nonautonomous dynamical sys-
tem in a Banach space.

**CORDERO, A.**

**Title:** Integrable Hamiltonian Systems on \(S^2 \times S^1\).
**Joint work with:** MARTÍNEZ ALFARO, J. and VIDEL, P.

**Abstract:** Casasayas et al. showed that Bott integrable Hamiltonian flows are closely related with
NMS flows. In fact, they prove that every 3-manifold being a non-singular constant-energy level
surface of a Bott integrable Hamiltonian systems, also holds a NMS flow. Also Campos showed that
when a given perturbation is applied on an integrable Hamiltonian field only remain periodic orbits
that are in the axis of the invariant tori or in the intersection of two tori. These periodic orbits are
called type NMS periodic orbits. So, the integrability of a Hamiltonian system can be inferred from
the periodic orbits obtained in the phase space and the links they become.

We will assume a Liouville integral Hamiltonian System with a second smooth integral which is
called a Bott integral \(f\). This is an integral whose critical points may be degenerate, but necessarily
organized as non-degenerate smooth manifolds. A smooth Bott integral cannot have isolated critical
points on a non-singular compact constant-energy level surface \(Q\); they fill either isolated smooth
critical circles or smooth two-dimensional tori.

Non orientable Bott integrals are admissible in a Hamiltonian system on \(Q = S^2 \times S^1\) and in
this case, Klein bottles \(K^2\) are found as critical submanifolds of \(f\) on \(S^2 \times S^1\). The NMS flow on
these Klein bottles is studied, and the possible periodic orbits on \(K^2\) are found. Nevertheless, it is
always possible to find an integrable Hamiltonian system on \(Q'\) with an orientable Bott integral \(f'\)
such that all critical Klein bottles on \(Q\) become in critical tori on \(Q'\). So, we only consider integrable
Hamiltonian systems with an orientable Bott integral.
We use the close relation between NMS flows and integrable Hamiltonian systems and the results previously obtained about the topological characterization of links of NMS periodic orbits on $S^2 \times S^1$ in order to obtain a complete characterization of knots and links of periodic orbits of integrable Hamiltonian systems on $S^2 \times S^1$.

**Keywords:** $S^2 \times S^1$, integrable Hamiltonian systems, NMS systems, round handle decomposition, links of periodic orbits.

DEL MAGNO, G.

**Title:** Bernoulli Elliptical Stadia II.

**Joint work with:** MARKARIAN, R.

**Abstract:** This talk complements Markarian’s talk. We generalize the results on the ergodicity and Bernoulli property presented in Markarian’s talk to a large class of billiard systems whose tables are bounded by focusing curves and, possibly, by dispersing curves and straight lines. An important step in the proof of the global ergodicity of a given non-uniformly hyperbolic system with singularities (billiards belong to such class of systems) is to establish that the system is locally ergodic (i.e., its ergodic components are open (mod 0)). Sinai first and then other authors proved that the local ergodicity of a non-uniformly system with singularities is verified if the system satisfies two properties: the non-contraction property and the so called Sinai-Chernov Ansatz. We sketch a proof of the non-contraction property for the class of billiards considered.

DJELLIT, A.

**Title:** Existence of Solutions for some Nonlinear Elliptic Systems.

**Joint work with:** TAS, S.

**Abstract:** We study a class of nonlinear elliptic systems involving the $p$-Laplacian. Under suitable assumptions on the nonlinearities, we show the existence of nontrivial solutions.

In this paper, we deal with the following nonlinear elliptic system which derives from a potential i.e. the nonlinearities on the right hand side are the gradient of a $C^1$-functional

\[
\begin{cases}
-\Delta_p u = \frac{\partial F}{\partial u}(x, u, v) \text{ in } \mathbb{R}^n \\
-\Delta_q v = \frac{\partial F}{\partial v}(x, u, v) \text{ in } \mathbb{R}^n
\end{cases}
\]  

(S)

where $\Delta_p u = \text{div}(|\nabla u|^{p-2}\nabla u)$ is the $p$-Laplacian, $1 < p, q < n$.

The operator called “$p$-Laplacian” appears in many nonlinear problems arising in physics, for example the flow of Non-Newtonian fluids correspond for $p > 2$ to dilatant fluids, for $1 < p < 2$ to pseudo-plastic fluids and the Laplacian ($p = 2$) to Newtonian fluids. Many authors studied the existence of solutions for such problems (equations or systems) for which explicit solutions generally can’t be given. In this work, we will show the existence of nontrivial solutions for System (S) in homogeneous Sobolev spaces under mixed subcritical growth conditions and the primitive $F(x, u, v)$ interacts with the first eigenvalue of an appropriate system. We use a variant of the Mountain Pass Theorem in which a weak version of the Palais-Smale condition, due to Cerami, holds.

DJELLIT, I.

**Title:** Homoclinic Bifurcations in a Two-Dimensional Endomorphism.

**Joint work with:** FERCHICHI, M.R.

**Abstract:** Our study concerns global bifurcations occuring in noninvertible maps, it consists to show that there exists a link between contact bifurcations of a chaotic attractor and homoclinic bifurcations of a saddle fixed point or a saddle cycle being on the boundary of the chaotic attractor.

We study particularly an example of bidimensional endomorphisms of $(Z_1 - Z_3 - Z_1)$ type. We will show that points of contact, between boundary of the attractor and its basin of attraction,
converge toward the saddle fixed point or the saddle cycle. These points of contact are also points of intersection between the stable and unstable invariant manifolds giving birth to homoclinic orbits (homoclinic bifurcations).

FABBRI, R.

Title: Linear Non-Autonomous Control Processes: Generalization of a Yakubovich’s Frequency Theorem and Criterion for the Absolute Stability.

Joint work with: IMPRAM, S., JOHNSON, R. and NÚÑEZ, C.

Abstract: Using methods of the theory of nonautonomous linear differential systems as the concepts of exponential dichotomy and rotation number we generalize some aspects of Yakubovich’s Frequency Theorem from periodic control systems to systems with bounded uniformly continuous coefficients and give a criterion for the absolute stability of such processes.

Keywords: Frequency Theorem, exponential dichotomy, rotation number, absolute stability.

FÉJOZ, J.

Title: Diffusion in the Planar Five-Body Problem.

Abstract: I will give a model of Arnold diffusion in the planar five-body problem. In this purpose I build some normal forms called the secular systems. I hope the global dynamics of these secular systems will show some drift behavior, along the lines of a geometric mechanism due to R. Moeckel.

Keywords: n-body problem, Arnold diffusion, non-integrability, secular systems.

GABERN, F.

Title: Restricted Four and Five Body Problems in the Solar System.

Joint work with: JORBA, À.

Abstract: We focus on the dynamics of a small particle near the Lagrangian points of the Sun-Jupiter system. To try to account for the effect of Saturn, we develop specific models based on the numerical computation of periodic and quasi-periodic (with two frequencies) solutions of the planar three-body problem for Sun, Jupiter and Saturn, close to the real motion of these three bodies. Then, we can write the equations of motion of a fourth infinitesimal particle moving under the attraction of these three masses. Using suitable coordinates, the model is written as a time-dependent perturbation of the well-known spatial Restricted Three-Body Problem. A model including Uranus is also developed.

Keywords: Three body problem, four body problem, invariant tori, Trojan asteroids.

GALÁN, J.

Title: Stability and Bifurcation Behavior of the Figure Eight Solution of the Three Body Problem.

Joint work with: MUÑOZ-ALMARAZ, F., FREIRE, E., DOEDE, E, VANDERBAUWHEDI, A.

Abstract: We analyze the bifurcation behavior of a recently discovered solution of the three body problem that in real space resembles the shape of an eight. By means of numerical continuation we investigate the local and global bifurcation behavior of this remarkable orbit and its connections to other solutions and to the restricted three body problem. We also address the problem of the stability properties of the real minimizer of the action by studying the second variation of the functional. This study is a benchmark for a more general scheme to continue periodic orbits and relative periodic orbits in symmetric Hamiltonian systems.

Keywords: Figure-8, three body problem.
GALPERIN, E.A.

**Title:** Some Generalizations of Lyapunov’s Approach to Stability and Control of Non-Linear Systems.

**Abstract:** Some new developments in Lyapunov’s approach are presented that include the generalized perturbation equation and its applications; the use of nonanalytic Lyapunov functions; extension of the Barbashin-Krasovskii theorem on asymptotic stability assured by a Lyapunov function with nonpositive derivative; the consistency condition for a time-space mosaic that constitutes a discontinuous Lyapunov function valid for investigation of stability; the introduction of non-sign definite functions for use in control (carrying surfaces); the extremal set construction for control, stabilization, and nonlinear asymptotic observer design.

GIDEA, M.

**Title:** A Topological Model for Arnold Diffusion.

**Joint work with:** ROBINSON, C.

**Abstract:** We propose a topological model for studying Arnold diffusion type of phenomena. Our approach is similar in spirit to Moeckel’s work on generic drifts on Cantor sets of annuli, but we are able to prove the existence of drifting orbits for particular systems. The technique we use is a refinement of Easton’s method of windows.

**Keywords:** Correctly aligned windows, symbolic dynamics, Arnold diffusion.

GIL’, M.

**Title:** Operator Functions in Theory of Nonlinear Continuous, Discrete and Retarded Dynamical Systems.

**Abstract:** We suggest a survey of some results on stability and periodic solutions of differential, difference and retarded equations in Euclidean and Banach spaces, which are based on recent estimates for norms of operator-valued functions.

One of the basic methods for investigation of solution stability is the Lyapunov functions (functionals) method by which many strong results are obtained. But finding Lyapunov functions is usually difficult. By the combined usage estimates for norms of operator-valued functions with the method of linearization and the freezing method new stability criteria are obtained. They make it possible to avoid the construction of Lyapunov’s functions in appropriate situations.

In our survey, we consider also a class of various classes of coupled systems of differential, neutral type FDEs as well as difference equations with separated linear terms and having nonlinearities with the Lipschitz property. We establish estimates for the Green functions of the linear terms. By these estimates, explicit conditions for the existence and uniqueness of periodic solutions are derived. The conditions are based on the spectral properties of the linear terms and the Lipschitz constants of the nonlinear terms. In addition, estimates for periodic solutions and their derivatives are proposed.

Some of the mentioned result are new, and some of them are taken from [1-4].

**References:**

GÓRNIEWICZ, L.

Title: On the Lefschetz Fixed Point Theorem (dedicated to Valter Seda on the occasion of his 70th birthday).

Abstract: We shall present current results concerning the Lefschetz Fixed Point Theorem for metric spaces. Some new results are included. In particular, an abstract version and also the Lefschetz Fixed Point Theorem for condensing mappings are proved. Finally, some possibilities of applications to differential equations will be signalize.

Keywords: Lefschetz number, fixed points, CAC-maps, condensing maps, ANR-spaces.

GRIGORIEVA, E.

Title: Attainable Sets of a Dynamical Microeconomic Model of the Process of Production, Storage, and Sales.

Joint work with: KHAIOV, E.

Abstract: In this work, a nonlinear model of the process of production, storage, and sales of a perishable consumer good has been simultaneously controlled by the rate of production and the price of the good. Attainable sets of controlled systems are investigated and the boundaries of these sets are studied. It is proved that only piecewise constant controls with at most two switchings can lead a trajectory to the boundary of the attainable sets. Shapes of attainable sets for different parameters of the model will be demonstrated using Maple.

Keywords: Attainable set, nonlinear controlled system, microeconomic dynamical model.

GUNESCH, R.

Title: Precise Asymptotics for Periodic Orbits of the Geodesic Flow.

Abstract: We establish the most precise asymptotic formula ever discovered for the number of periodic orbits for the geodesic flow on a rank one manifold of nonpositive curvature (counted by homotopy). This extends a celebrated result of Fields medalist G.A. Margulis to the nonuniformly hyperbolic case and strengthens previous results by G. Knieper.

While proving this result, we also manage to carry out Margulis’ construction of the measure of maximal entropy without requiring strong hyperbolicity.

HARO, A.

Title: Invariant Manifolds in Quasiperiodic Systems: Theory, Computation and Applications.

Joint work with: DE LA LLAVE, R.

Abstract: We explain the parameterization method to prove the existence of (normally hyperbolic) invariant manifolds in quasi-periodic systems. It also provides an effective algorithm to compute these manifolds. We put special emphasis on Hamiltonian systems, applying the computer methods to the quasiperiodic Hénon map and the quasiperiodic standard map.

Keywords: Quasiperiodic systems, normal hyperbolicity, invariant tori, invariant manifolds, computation.

HORITA, V.

Title: Non-Periodic Bifurcations of One-Dimensional Maps.

Abstract: We construct an open class of 2-parameter families of 1-dimensional maps exhibiting, for a large set of parameters, non-periodic bifurcation on the boundary of hyperbolic maps with all periodic points being hyperbolic. Moreover, for these maps we construct an absolutely continuous invariant measure. It can be carry out from a work of Newhouse-Palis-Takens the conjecture that generically maps in the boundary of hyperbolic maps have a periodic bifurcation (they prove this statement when the limit set is finite). In some measure theoretical sense, we present a large (positive
measure) subset of the boundary of hyperbolic maps with non-periodic bifurcation. We also provide information about the boundary of hyperbolic maps.

JODAYREE AKBARFAM, A.

Title: On the Higher-Order Asymptotic Distributions of the Eigenvalues for Sturm-Liouville Problems with Different Types of One Turning Point.

Joint work with: MINGARELLI, A.

Abstract: In this paper, we study the higher order asymptotic distribution of the real eigenvalues associated with a linear real second order equation

\[ y'' + (\lambda r(x) - q(x))y = 0, \quad a \leq x \leq b, \]

with Dirichlet boundary conditions, where \( r(x) \) has one zero at \( x = x_\nu \) of order \( \ell \), and \( q \) is a real-valued function in \( L^1(a,b) \). This extends previous work. The turning point was considered in different types, in particular, when one of the end points is a turning point.

This work, was completed while the first author was visiting Carleton University in 2000-2001.

KIRIKI, S.

Title: Parameter-Shifted Shadowing Property for Lorenz Attractors.

Joint work with: Soma, T.

Abstract: By Tucker’s study, we have known that the Lorenz system has a robust strange attractor for Lorenz’ original parameter, which is constructed by the same rule of the geometric Lorenz model given by Guckenheimer and others. We now focus some shadowing property of the geometric Lorenz model.

There is an old work about a shadowing property for the geometric Lorenz family given by Komuro. However, in his scope of the shadowing property, the geometric Lorenz family can never have the shadowing property excepting a finite-time case or some special geometric settings. By introducing more suitable scope of the shadowing property which is called parameter-shifted, we present new results for the same theme in [1]. First, we introduce our parameter-shifted shadowing property, abbreviated to “PSSP”, to some set \( \mathcal{L} \) of Lorenz maps on the plane. By applying the similar method which is used in our another study of the PSSP for Lozi maps [2], we get the first result:

**Theorem 1 (PSSP for the Lorenz map).** There exists an open set \( \mathcal{O} \subset \mathcal{L} \) such that every Lorenz map \( L \in \mathcal{O} \) has the parameter-shifted shadowing property.

Let us consider a set \( \mathcal{X}_\mathcal{O} \) of vector fields generated from the geometric Lorenz method controlled by Lorenz maps in \( \mathcal{O} \). We introduce two concepts weak and strong parameter-shifted shadowing property to flows of such vector fields. The main results are as follows:

**Theorem 2 (Weak PSSP for the Lorenz flow).** Every geometric Lorenz flow generated by a vector field in \( \mathcal{X}_\mathcal{O} \) has the weak parameter-shifted shadowing property.

**Theorem 3 (Strong PSSP for the Lorenz flow).** Every geometric Lorenz flow has no strong parameter-shifted shadowing property.

**Keywords:** Geometric Lorenz family, Lorenz map, shadowing property.

**References:**


KOTUS, J.

Title: Geometric and Invariant Measures for Transcendental Functions.

Joint work with: URBAŃSKI, M.

Abstract: We consider the problem of the existence of absolutely continuous invariant measure for transcendental meromorphic functions. We prove sufficient conditions for a subexpanding meromorphic function $f$ to have $\sigma$-finite absolutely continuous invariant measure $\mu$ defined on the Julia set $J(f)$, i.e.

**Theorem 1.** Let $f : \mathbb{C} \to \overline{\mathbb{C}}$ be a transcendental meromorphic function satisfying the following conditions:

(a) $J(f) = \overline{\mathbb{C}},$
(b) $l_2(I_\infty(f)) = 0 \{I_\infty(f) = \{z : f^n(z) \to \infty\}\},$
(c) $l_2(\{z : \omega(z) \subset P(f)\}) = 0.$

Then there exists a $\sigma$-finite $f$-invariant measure $\mu$ equivalent with the Lebesgue measure $l_2.$

We also describe the fractal structure of expanding meromorphic maps of the form $f(z) = H \circ \exp \circ Q,$ where $H$ and $Q$ are rational functions whose most transparent examples are among the functions of the form $A \exp(z^p) + B \exp(-z^p)$ with $AD - BC \neq 0.$ In particular we show that depending up on whether the Hausdorff dimension $h$ of the Julia set $J(f)$ is greater or less than 1 the finite non-zero geometric measure is provided by the Hausdorff $H^h$ or packing measure $P^h,$ i.e.

**Theorem 2.** If the Julia set $J(f)$ is hyperbolic, then

(a) if $h < 1,$ then $0 < P^h(J(f)) < \infty$ and $H^h(J(f)) = 0,$
(b) if $h = 1,$ then $0 < P^h(J(f)), H^h(J(f)) < \infty,$
(c) if $h > 1,$ then $0 < H^h(J(f)) < \infty$ and $P^h(J(f)) = \infty,$

and there exists a unique probabilistic invariant measure equivalent to a geometric conformal measure.

In order to describe this fractal structure we introduce and explore in detail Walters expanding conformal maps and jump-like conformal maps.

LENBURY, Y.

Title: Identification of Limit Cycle Behavior in Higher Dimensional Cascade Systems by Separation Conditions: Application to Communities Coupled by Migration.

Joint work with: DUMRONGPOKAPHAN, T. and CROOKE, P.S.

Abstract: In this paper, explicit separation conditions are derived for the identification of limit cycle behavior in a higher dimensional (4 $\leq n$) cascade system. The system is assumed to be characterized by highly diversified dynamics which allows for the use of singular perturbation principles, based on simple geometric arguments. The conditions for the identification of relaxation oscillation in slow-fast system will be extended to accommodate dynamical systems in which more state variables are involved in multiscale interactions. We then demonstrate the use of the conditions thus derived by applying the principle to a model of two communities coupled by migration, leading to valuable insights into how the mechanism of migration may be exploited to ensure survival and coexistence of the interacting species in the eco-system.

Keywords: Limit cycles, singular perturbation, dynamical systems, migration in eco-systems.

LOPES-DIAS, J.

Title: Renormalization of Vector Fields.

Abstract: We use a renormalization operator $R$ acting on a space of vector fields on the multidimensional torus $\mathbb{T}^d,$ $2 \leq d,$ to prove the existence of a local submanifold of vector fields equivalent to constant. The result comes from the existence of a fixed point $\omega$ of $R$ which is hyperbolic. This is done for a certain class $KT_{\omega}$ of frequency vectors $\omega,$ called of Koch type. The transformation $R$ is
constructed using a time rescaling, a linear change of basis plus a periodic non-linear map isotopic to the identity, which we derive by a “homotopy trick”.

This idea is then used to construct rigorous renormalization schemes for Poincaré vector fields on \( \mathbb{T}^2 \), and to determine existence of invariant tori for various flows with extra vertical directions. We show that iterating this procedure there is convergence to a limit set with a “Gauss map” dynamics on it, related to the continued fraction expansion of the slope of the frequencies. This is valid for diophantine frequency vectors. Applications to Hamiltonian systems are discussed.

MALEKNEJAD, K.

Title: Solution of Differential Equations via Wavelet Functions.

Joint work with: MESGARANI, H.

Abstract: Wavelet functions are developed to approximate the solution of the differential equations. Properties of wavelet functions are first presented, the operational matrix of integration together with product operational matrix are utilized to reduce the computation of differential equations. The method is computationally attractive, and applications are demonstrated through illustrative examples.

Keywords: Wavelet functions, algebraic equations, orthogonal functions.

MARZOUGUI, H.

Title: Area Preserving Flows and Metrical Transitivity.

Abstract: M.D. Hirsch has shown in [1] that an area-preserving flow \( \Phi \) on a closed surface \( M \) is a topologically transitive if it has no periodic orbits and the graph of singular orbits does not separate \( M \). The aim of this communication is to give a simple proof of this result and to get an improvement for the questions given in [1] as mentioned in Theorem 2.

Let \( \Phi \) be an area preserving flow on \( M \). We say that \( \Phi \) is metrically transitive if for every \textit{closed} invariant set \( A \), the Lebesgue measure of \( A \) is 0 or 1. For this weaker definition of metric transitivity, it is well known that if \( \Phi \) is metrically transitive then it is topologically transitive. In [2], it was established the following converse:

\textbf{Theorem 1} [2]. Let \( \Phi \) be a flow with a compact countable set of singularities on a closed orientable surface \( M \). If \( \Phi \) is topologically transitive then it is metrically transitive.

We prove the following:

\textbf{Theorem 2.} Let \( \Phi \) be an area preserving flow whose set \( \text{sing}(\Phi) \) of singularities is a compact totally disconnected set on \( M \). Let \( D \) be the set of dense orbits of \( \Phi \). If the flow \( \Phi \) is topologically transitive, then:

i) if \( D \) is full measure the flow \( \Phi \) is metrically transitive;

ii) if \( \text{sing}(\Phi) \) is countable then \( D \) is full measure.

These results are recent and will appear in [3], issue september 2002.

Keywords: Flows on surfaces, area preserving flows, lower structure, orbit, topological transitivity, metrical transitivity.

References:


MAZUR, M.

Title: Shadowing and Likes as \( C^0 \) Generic Properties.

Abstract: We review a concept of shadowing, inverse shadowing and tolerance stability. We also announce some new results in this direction, among them the solution of the main part of Zeeman’s
Tolerance Stability Conjecture.

Keywords: (Weak, inverse) shadowing, pseudo-trajectory, $C^0$ generic property, (strong) tolerance stability, Zeeman’s Tolerance Stability Conjecture, chain recurrence.

MOSHCHEVITIN, N.G.

Title: Recurrence of Solutions of Differential Equations with Almost- and Quasiperiodic Coefficients.

Abstract: We consider a linear differential equation $dX/dt = A(t)X$ where $X$ is an orthogonal matrix and skew-symmetric smooth matrix $A(t)$ depends on the time in the almost-periodic way. We prove the recurrence property for any particular solution and in the case of the finite basis of frequencies we give a quantitative recurrence result. We discuss the uniform recurrence. If the equation under consideration can be reduced by a linear quasiperiodic matrix to an equation with constant coefficients it is easy to observe the uniform recurrence property with ”power order”. By means of a number-theoretical and Diophantine approach we can construct analytic systems without such type of recurrence. Hence these equations are irreducible.

MOZO, J.

Title: Summability for Singularly Perturbed Differential Equations.

Abstract: In this talk we study the summability properties of the formal power series solutions of holomorphic singularly perturbed differential equations $\varepsilon^h x^{k+1} y' = f(x, y, \varepsilon)$, defined in a neighbourhood of the origin. When $k = -1$, it is known that solutions are not, in general, summable. If $k = 0$, there is not in general a formal solution. If $k > 1$, under suitable conditions of nonsingularity of the equation, formal solutions in both variables $\varepsilon$ and $x$ exist, and are of Gevrey type. After defining a good notion of summability for series of this kind, generalizing the good notion of the single variable case, we shall show that this series are summable in this sense, when the equation is linear. These are results obtained in collaboration with Werner Balser (Univ. of Ulm). We shall try to generalize these results to the nonlinear case.

Keywords: Asymptotic developments, singular perturbations, summability.

NERURKAR, M.

Title: On a Conjecture of A. Forrest.

Abstract: Consider the skew-shift $T(x, y) = (x + \alpha, x + y)$ on the 2-torus, where $\alpha$ is irrational. Consider the cocycle $(x, y) \rightarrow e^{iy}$ from the 2-torus to the additive group of complex numbers and form the skew-product transformation $\hat{T}(x, y, z) = (x + \alpha, x + y, z + e^{iy})$. A. Forrest proved that if $\alpha$ is sufficiently rapidly approximable by rationals then $\hat{T}$ is point transitive and conjectured that it must be ergodic for some irrationals. We construct irrationals for which this conjecture is true. The construction involves use of certain estimates of Hardy and Littlewood along with Khinchine’s results from the metric theory of diophantine approximation.

NIEDERMAN, L.

Title: Exponential Stability for Small Perturbations of Steep Integrable Hamiltonian Systems.

Abstract: In the 70’s, Nekhorchev proved that for an analytic nearly integrable Hamiltonian system with a perturbation of size $\varepsilon$, the actions linked to the unperturbed Hamiltonian vary only by the order of $\varepsilon^b$ over a time of the order of $\exp (C \varepsilon^{-a})$ for some positive constants $a$, $b$ and $C$, provided that the unperturbed Hamiltonian meets some generic transversality condition known as steepness. Among steep systems, convex or quasiconvex systems are easier to describe since the use of energy conservation allows to shorten the proof of exponential estimates of stability. In this case, Lochak and Pöschel have independently obtained the stability exponents $a = b = 1/2n$ for systems of $n$ degrees of freedom - especially the time exponent $(a)$ is expected to be optimal. Moreover, the study of Lochak relies on simultaneous Diophantine approximation which gives a very transparent
On the other hand, the proof in the steep case has almost not been taken up since Nekhorochev's original work despite various physical examples where the model Hamiltonian is only steep. Here, we combine the original scheme with simultaneous Diophantine approximation as in Lochak's proof. This yields significant simplifications with respect to Nekhorochev's reasoning, it also allows to obtain the exponents $a = b = (2np_1 \ldots p_n)^{-1}$ where $(p_1 \ldots p_n)$ are the steepness indices of the considered Hamiltonian. In the quasiconvex case the steepness indices are all equal to one, and we find the same exponents $1/2n$ as Lochak and Pöschel, whose results are thus generalized in the steep case.

Finally, in a recent work with J. Féjoz, we related the steepness indices to the Łojaciewicz's exponents and give methods to compute these indices in a general setting.

OLEK, A.

**Title:** Continuous Dependence on Obstacles in Double Global Obstacle Problems.

**Joint work with:** SZCZEPANIAK, K.

**Abstract:** Let $\Omega \in \mathbb{R}^n$ be bounded domain with smooth boundary, $A: H^1_0(\Omega) \to H^{-1}(\Omega)$ be Lipschitz and coercive operator. Let $\langle \cdot, \cdot \rangle$ stand for the dual bracket between $H^1_0(\Omega)$ and $H^{-1}(\Omega)$. For the given functions $\varphi, \psi$ we define (under suitable assumptions) the admissible set $K_{\psi}^\varphi := \{ \nu \in H^1_0(\Omega) : \varphi \leq \nu \leq \psi \text{ a.e. in } \Omega \}$. Then for $f \in H^{-1}(\Omega)$ we discuss the following obstacle problems:

- $(P_n)$: Find $u_n \in K_{\psi}^\varphi : \langle A u_n, \nu_n - u_n \rangle \geq \langle f, \nu_n - u_n \rangle \forall \nu_n \in K_{\psi}^\varphi$.
- $(P)$: Find $u \in K_{\varphi}^\psi : \langle A u, \nu - u \rangle \geq \langle f, \nu - u \rangle \forall \nu \in K_{\varphi}^\psi$.

The purpose of this work is to study convergence (in Mosco sense) of the solutions $u_n$ of $(P_n)$ providing the sequences of impediments converge. Such problems are known as “varying obstacle problems”.

PANEAH, B.

**Title:** Dynamical Approach in Analysis: Functional Equations, Boundary Problems for PDE, Integral Geometry.

**Abstract:** This talk is devoted to solution of several new problems in the three independent fields of Analysis: functional equations, boundary problems for higher order hyperbolic differential equations in bounded domains and integral geometry. All these problems at first sight do not give even a merest hint about some dynamical systems connected with them. Nevertheless it turned out that when solving these quite different problems an essential part of information can be obtained with the help of dynamical methods. To apply these methods we introduce a semigroup $\Phi$ of maps in an interval $I$ generated by two maps $\alpha$ and $\beta$ in $I$ which are closely connected with the problems in question. On a side the language of orbits of this semigroup enables to formulate easily conditions (sometimes necessary and sufficient) of solvability of these problems. On the other side one of the most essential technical elements in the proof of the main statements is a searching of some specific attractors of the noncommutative dynamic system generated by semigroup $\Phi$. Even in functional equations with their long history our approach allows to obtain completely new results which have nothing in common with what was known earlier. And this is without any hard analytical work. In particular we solve at the first time a nonhomogeneous Cauchy equation on a curve and improve significantly what was known about homogeneous one.

Note that purely dynamical part of the talk seems to contain setting of new problems related to noncommutative dynamical systems.

PEIRONE, R.

**Title:** Homogenization of Linear Transport Equations.

**Abstract:** I discuss the homogenizability of linear transport equations with periodic data. It can be interpreted in terms of dynamical system properties of the associated ODEs, in some sense it is related to the rotation vectors of such an equation. E. De Giorgi conjectured that all equations of
this kind are homogenizable. The results are that we have in fact homogenizable two-dimensional
case for nonvanishing functions (partial results in this direction have been previously proved by other
authors). On the other hand, in the three-dimensional case, we have an example of nonhomogenizable
equation.

**PONCE, E.**

**Title:** Limit Cycle Bifurcation from Infinity in Symmetric 3D Piecewise Linear Systems.

**Joint work with:** FREIRE, E. and ROS, J.

**Abstract:** Piecewise linear systems are of great interest in engineering applications, for instance
in nonlinear control and electronics. Even for continuous vector fields, their non-smooth character
enforces the development of new practical tools for bifurcation analysis. We will consider the most
frequent case, i.e. continuous symmetric systems whose fundamental nonlinearity is made up of
three linear pieces. Regarding the 2D case, it has been possible to adapt with some effort certain
planar techniques to cope with Hopf bifurcations at infinity, see [1]. However, 3D piecewise linear
systems are more difficult to deal with. The study of the so-called closing equations constitutes
a technique well suited to piecewise linear systems for the analysis of the focus-center-limit cycle
bifurcation, see [2] for the planar case. This technique has no dimension limitations, but it requires
some adaptations to capture limit cycles of great amplitude. We will show how it is possible to
use the closing equations in order to give a complete characterization of the 3D Hopf bifurcation
at infinity, gaining information about criticality (as given in [3]) and also about the unfolding of
possible degeneracies, along with quantitative approximations to the period, the amplitude and the
characteristic exponents of the bifurcating limit cycle.

**Keywords:** Limit cycles, Hopf bifurcations, piecewise linear systems.

**References:**
1. LLIBRE, J., PONCE, E., Bifurcation of a periodic orbit from infinity in planar piecewise linear
2. FREIRE, E., PONCE, E., ROS, J., Limit cycle bifurcation from center in symmetric piecewise-
3. DIAMOND, P., KUZNETSOV, N., RACHINSKI, D., On the Hopf bifurcation in control systems
   with a bounded nonlinearity asymptotically homogeneous at infinity, *J. Differential Equations* 175

**PUIG, J.**

**Title:** Hill’s Equation with Quasi-Periodic Forcing: Resonance Tongues and Reducibility.

**Joint work with:** SIMÓ, C.

**Abstract:** In this talk we shall present some results on Hill’s equation with quasi-periodic forcing,
namely

\[ x'' + \left( a + \sum_{i=1}^{p} b_i q_i(t) \right) x = 0, \quad (1) \]

where \( \{a, b\} \in \mathbb{R}^{p+1} \) are parameters and \( q_i \) are analytic quasi-periodic with Diophantine frequency.
For a fixed value of \( b \in \mathbb{R}^p \) this is Schrödinger equation with quasi-periodic potential, which has
been extensively studied. On the other hand, equation (1) is a generalization of Hill’s equation
with periodic forcing, for which Floquet theory of reducibility of periodic systems provides a tool to
study the stability and instability zones (called resonance tongues) in the parameter space. In the
quasi-periodic case we do not have such a theory, but we can still define resonance tongues using
the concept of the rotation number of (1). The analyticity of these tongue boundaries, which in the
periodic case is a trivial consequence of Floquet’s theory, turns out to be a problem of reducibility,
involving small divisors and inductive schemes, which can be formulated in a general setting in order
to be able to handle with different situations.
**Keywords:** Hill’s equation with quasi-periodic forcing, Schrödinger equation with quasi-periodic potential, rotation number, Floquet theory, reducibility of linear quasi-periodic systems, resonance tongues, small divisors.

**RIAZA, R.**

**Title:** Multiple Singular Bifurcation in Matrix Pencils and DAEs.

**Abstract:** The singularity-induced bifurcation theorem has been recently proved for parameterized semiexplicit index-1 differential-algebraic equations (DAEs) modelling power systems [1, 4, 5]. The SIB Theorem describes the divergence of one eigenvalue through infinity when an equilibrium curve crosses a manifold of singularities. Recent results place this issue in a matrix pencil context, showing that this phenomenon follows from a minimal index change at the singularity, and providing an extension to quasilinear implicit ODEs and also to higher index DAEs in Hessenberg form [2, 3]. In the present communication, we extend this approach to cover situations in which several eigenvalues diverge. Specifically, we prove that a minimal rank (resp. index) change allows one to compute the number of diverging eigenvalues in terms of an index (resp. rank) change in the pencil.

**References:**

**SANZ, A.M.**

**Title:** Almost Periodic and Almost Automorphic Dynamics for a Class of Scalar Convex Differential Equations.

**Joint work with:** NOVO, S.

**Abstract:** We study the variation of the set of bounded trajectories of the flow defined by a certain class of scalar convex differential equations depending on a parameter. It is found that there exists precisely one value of the parameter for which almost automorphic but not almost periodic dynamics may appear. Even for this parameter value, the occurrence of almost automorphic dynamics is shown to be unusual in a precise topological sense.

**SAPRYKINA, M.**

**Title:** Some New Examples in Analytic Ergodic Theory.

**Abstract:** We present a construction method permitting to obtain examples of non-linearizable Lebesgue measure preserving analytic diffeomorphisms of the two-torus, enjoying different statistical properties with respect to this measure: minimal and non-ergodic, uniquely ergodic, weakly mixing. Construction of weakly mixing diffeomorphisms is a joint work with B. Fayad.

**SIRVENT, V.F.**

**Title:** Pure Discrete Spectrum for One-Dimensional Substitution Systems of Pisot Type.

**Joint work with:** SOLOMYAK, B.

**Abstract:** We consider two dynamical systems associated with a substitution of Pisot type: the usual Z-action on a sequence space, and the R-action, which can be defined as a tiling dynamical system or as a suspension. We describe procedures for checking when these systems have pure discrete spectrum (the “balanced pairs algorithm” and the “overlap algorithm”) and study the relation between them. In particular, we show that pure discrete spectrum for the R-action implies pure...
discrete spectrum for the $Z$-action, and obtain a partial result in the other direction. As a corollary, we prove pure discrete spectrum for every $\mathbb{R}$-action associated with a two-symbol substitution of Pisot type (this is conjectured for an arbitrary number of symbols).

**Keywords:** Pure discrete spectrum, tiling dynamical systems, substitutions dynamical systems.

ŠKAPIN-RUGELJ, M.

**Title:** Symbolic Dynamics in Investigation of Quaternionic Julia Sets.

**Joint work with:** PETEK, P. and LAKNER, M.

**Abstract:** We introduce symbolic dynamics regarding the sign of the real component of the quaternion point in the Julia set. Points with a given itinerary constitute smooth curves, the Julia set however has a fractal structure. Of special interest are repeating itineraries and periodic points.

**Keywords:** Dynamics, quaternion, periodic points, Julia set.

TAGHAVI, A.

**Title:** On Periodic Solutions of Liénard Equation.

**Abstract:** It is conjectured by Pugh, Lins, and de Melo in [1] that the system of equations

$$\begin{cases} \dot{x} = y - F(x) \\ \dot{y} = -x \end{cases}$$

has at most $n$ limit cycles when the degree of $F = 2n + 1$ or $2n + 2$. Put $M$ for uniform upper bound of the number of limit cycles of all systems of equations of the form

$$\begin{cases} \dot{x} = y - (ax^4 + bx^3 + cx^2 + dx) \\ \dot{y} = -x \end{cases}$$

In this article, we show that $M \neq 2$. In fact, if an example with two limit cycle existed, one could give not only an example with $n + 2$ limit cycles for the first system, but also one could give a counterexample to the conjecture $N(2,3) = 2$ [see the conjecture $N(2,3) = 2$ of F. Dumortier and C. Li: Quadratic Liénard equation with Quadratic Damping, *J. Differential Equations* 139 (1997), 41–59]. We will also pose a question about complete integrability of Hamiltonian systems in $\mathbb{R}^4$ which naturally arise from planner Liénard equation. Finally, considering the Liénard equation as a complex differential equation, we suggest a related problem which is a particular case of conjecture. We also observe that the Liénard vector fields have often trivial centralizers among polynomial vector fields.

**Keywords:** Limit cycles, Liénard equation.

**References:**

TANIGUCHI, T.

**Title:** The Exponential Stability for Navier-Stokes Equation with Time Delay External Force.

**Abstract:** Let $\Omega$ be a regular open domain of $\mathbb{R}^2$ with boundary $\Gamma$. We are concerned with the stability of solutions to the following 2D-Navier-Stokes equations with the time delay external force field:

$$\begin{align*}
\frac{dx(t)}{dt} &= [\nu \Delta x(t) - (x(t), \nabla) x(t) + f(t - r, x(t - r)) + \nabla p]dt , \\
\text{div}(x) &= 0 \quad \text{in} \quad [0, \infty) \times D , \\
x(t, z) &= 0 \quad \text{on} \quad [0, \infty) \times \Gamma , \\
x(s, z) &= \varphi(s) \quad (s, z) \in [-r, 0] \times D ,
\end{align*}$$

where $r > 0$, $x$ is the viscosity field of the fluid, $p$ the pressure, $\nu > 0$ the kinematic viscosity, $\varphi$ the initial vector field, $f$ the external force field.
In this talk we discuss the exponential stability of weak solutions to Navier-Stokes equations with the time delay external force \( f(t-r, x(t-r)) \), where \( |f(t, x)| \leq a |x|^k + b, \ a, b \geq 0, \ k \geq 2 \). We show the relations between the kinematic viscosity \( \nu \) and the numbers \( a, b, k \) play an important role for the criterion on the stability of solutions.

**Keywords:** Exponential stability, weak solution.

**TATJER, J.C.**

**Title:** Codimension-Two Homoclinic Bifurcations in Two-Dimensional Non-Invertible Quadratic Maps.

**Abstract:** We study a certain family of two-dimensional non-invertible quadratic maps that can be considered as a model of the return map near a codimension-two homoclinic bifurcation of a three-dimensional dissipative saddle with real eigenvalues that we call generalized homoclinic tangency, see *Ergod. Th. Dynam. Sys.* **21** (2001), 249–302. We adapt the definition to the two-dimensional non-invertible case, and prove that the quadratic family has this type of bifurcation. This allows us to prove that there are infinitely many values of the parameters having Hopf and Bogdanov-Takens bifurcations, and therefore attracting closed invariant curves, and strange attractors. As all the computations are explicit, we can numerically compute bifurcation curves, and get a more global behaviour of this family.

**Keywords:** Strange attractors, closed invariant curves.

**TERUEL, A.E.**

**Title:** Limit Cycles for Planar Piecewise Linear Differential Systems.

**Joint work with:** NÚÑEZ, E. and LLIBRE, J.

**Abstract:** The existence, number and localization of limit cycles is one of the most important problems in the qualitative theory of planar differential equations, and the more usual tool for studying it is the Poincaré map. In the particular case of planar piecewise linear differential systems of the form

\[
\dot{x} = A x + \varphi(k^T x) b + a,
\]

where \( A \) is a \( 2 \times 2 \) real matrix, \( x, a, b, k \in \mathbb{R}^2 \), and \( \varphi \) is a continuous piecewise linear function, the Poincaré map is defined by the flow of the system when we take as transversal sections the boundaries of the regions where the system is linear.

For the piecewise linear differential systems having continuous characteristic functions \( \varphi \) formed by two-pieces of linear functions or by three-symmetric-pieces of linear functions, this problem has been completely solved, see [2] and [3]. But this technique becomes difficult to handle if we add more pieces or if we avoid the symmetry of \( \varphi \).

Using the Darboux theory of integrability and the election of an adequate coordinate system we provide an alternative way to the Poincaré map for analyzing the limit cycles of planar piecewise linear differential systems.

**Keywords:** Limit cycles, first integrals, piecewise linear systems.

**References:**


VALDINOCI, E.


Abstract: We consider functionals related with fluid jets and Ginzburg-Landau models in periodic media. We show that there exists a universal constant $C$ so that, given any plane $\pi$, there exists a minimal solution $u_\pi$ whose level sets are in a strip of width $C$ around $\pi$.

Keywords: Plane-like minimizers, jet flows, Ginzburg-Landau.

VALERO, J.

Title: Asymptotic Behaviour of a Partly Dissipative Reaction-Diffusion System.

Abstract: In this work we study the existence and asymptotic behaviour of solutions of the system of partly dissipative reaction-diffusion equations

$$
\begin{align*}
\frac{\partial u}{\partial t} - \Delta u + f_1(u) - f_2(u) &\ni h_1(u, v) + d_1(t), \\
\frac{\partial v}{\partial t} + \sigma v &= h_2(u) + d_2(t), \quad \text{in (0, T) \times } \Omega, \\
u |_{\partial \Omega} &= 0, \\
u |_{t=0} &= u_0, \quad v |_{t=0} = v_0,
\end{align*}
\tag{1}
$$

where $f_i$ are maximal monotone functions (may be discontinuous). A particular case of this system is a model for the conduction of electrical impulses in a nerve axon.

In [1] some abstract results on the existence of solutions of abstract differential inclusions generated by a difference of subdifferential maps are given. These theorems allow to obtain the existence of solutions of the preceding models in several cases. Using the results of [1], in the paper [2] it is studied the asymptotic behaviour of the solutions to inclusion (1) in the case where we have only the first equation (corresponding to $u$) and $f_2$ has at most linear growth.

Now we obtain the existence of strong (globally defined) solutions to (1) imposing a growth condition on $f_2$ which is weaker than the linear one. Then we construct a multivalued semiflow (the generalization of a semigroup for equations without uniqueness) and prove the existence of a global compact attractor.

Keywords: Partly dissipative reaction-diffusion equations, global attractor, differential inclusions, asymptotic behaviour.

References:
[1] Otani, M., On existence of strong solutions for $\frac{du}{dt} + \partial \psi_1(u(t)) - \partial \psi_2(u(t)) \ni f(t)$, J. Fac. Sci. Univ. Tokio. sect. IA Math. 24 (1977), 575–605.

VILLANUEVA, J.

Title: Asymptotic Size of Herman Rings of the Complex Standard Family.

Joint work with: FAGELLA, N. and MARTÍNEZ-SEARA, T.

Abstract: Let $G(z) = ze^{i\omega \varepsilon^2}$ denote the semistandard map, with $\omega \in \mathcal{B}$, where $\mathcal{B}$ denotes the set of Brjuno numbers in $\mathbb{R} \setminus \mathbb{Q}$. For this map, $z = 0$ is a fixed point with derivative $e^{i\omega}$ and $G$ has a Siegel disc $U$ around this point. Let $R$ denote the conformal radius of $U$, that is, the radius of the maximal disc where the linearization map is defined.

On the other hand we consider the complexification of the standard family of Arnold maps given by $F_{\alpha, \varepsilon}(z) = \alpha(\varepsilon^{\frac{\alpha(\varepsilon)^2}{2}}, z)$, with two real parameters $\varepsilon \in (0, 1)$ and $\alpha = \alpha(\varepsilon) \in [0, 2\pi)$ chosen so that $F_{\alpha, \varepsilon}$ restricted to the unit circle has rotation number $\omega$. Taking $\varepsilon$ small enough, it is known that a Herman ring $U_\varepsilon$ exists around the unit circle for $F_{\alpha, \varepsilon}$. Let $\hat{R}_\varepsilon$ denote the conformal size of the Herman ring, that is to say that the linearization map is defined on an annulus $\{1/\hat{R}_\varepsilon < |z| < \hat{R}_\varepsilon\}.$
Note that $R_\varepsilon$ tends to infinity as $\varepsilon$ tends to infinity. More precisely, estimates in [2] applied to the standard family give $R_\varepsilon > 2K/\varepsilon$, where $K$ depends only on the Brjuno number $\omega$.

The complex standard family is usually studied (for $\varepsilon$ small) as a perturbation of the rigid rotation. In this work, we investigate $F_{\alpha,\varepsilon}$ as a perturbation of the semistandard map, as seen from the expression $F_{\alpha,\varepsilon}(z) = ze^{i\alpha}e^{-\varepsilon z^2}$, which is obtained from $F_{\alpha,\varepsilon}$ after a change of variables $z = \frac{\varepsilon}{\varepsilon^2}u$. Changing variables also in the linearization plane, the Herman ring is the image, under the linearizing map, of an annulus $\{ |z| < R_\varepsilon \}$ where $R_\varepsilon = \frac{\varepsilon}{\varepsilon^2}R_\varepsilon^*$. Using a very explicit quasiconformal surgery construction to relate $F_{\alpha,\varepsilon}$ to $G$, we are able to relate the conformal radius $R$ of the Siegel disc, to the size $R_\varepsilon$ of the scaled Herman ring. More precisely, we prove that $R_\varepsilon = R + O(\varepsilon \log(\varepsilon))$, which translated to the original variables gives $R_\varepsilon = \frac{\varepsilon}{\varepsilon^2}(R + O(\varepsilon \log(\varepsilon)))$.

References:

WILCZAK, D.

Title: Heteroclinic and Homoclinic Connections and Symbolic Dynamics in Planar Restricted Circular Three Body Problem.

Joint work with: ZGLICZYŃSKI, P.

Abstract: The restricted circular three body problem is considered for the following parameter values $C = 3.03$ and $\mu = 0.0009537$ - the values for the Oterma comet in the Sun-Jupiter system. In [1] a very good numerical evidence was given for the following facts for the Sun-Jupiter-Oterma system:

1. the existence of Lyapunov orbits $L_1^*$ and $L_2^*$ around libration points $L_1$ and $L_2$, respectively,
2. the existence of transversal heteroclinic orbits connecting $L_1^*$ and $L_2^*$ in both directions,
3. the existence of a transversal homoclinic orbit to $L_1^*$ in the interior (Sun) region and a transversal homoclinic orbit to $L_2^*$ in the exterior region.

We present [2] a computer assisted proof of the existence of heteroclinic cycle between two Lyapunov orbits and the existence of homoclinic orbits to Lyapunov orbits $L_1^*, L_2^*$ - in exterior and interior regions. Moreover, we prove the existence of symbolic dynamics on six symbols built on heteroclinic and homoclinic orbits.

References:

[http://www.im.uj.edu.pl/~zgliczyn]

WÓJCIK, K.

Title: Topological Horseshoes and Delay Differential Equations.

Joint work with: ZGLICZYŃSKI, P.

Abstract: Consider a delay differential equation and an ordinary differential equation

$$x'(t) = f(x(t - \tau)), \quad \text{(1)}$$
$$x' = f(x), \quad \text{(2)}$$

where $x \in \mathbb{R}^n$, and $f \in \mathcal{C}^1$.

We study the following question: Assume that equation (2) has chaotic solutions, i.e. a suitable Poincaré map has a Smale’s horseshoe. Will the corresponding delay equation (1) also have chaotic solutions for small delays $\tau > 0$?
We present a proof that the answer to this question is positive, namely for small delays a suitable Poincaré map, $P_\tau$, for delay equation does have a topological horseshoe. We prove that this implies an existence of an invariant set $S_\tau$ for $P_\tau$, such that $P_\tau$ on $S_\tau$ can be semiconjugated with a Bernoulli shift. Moreover we obtain an infinite number of periodic orbits with unbounded periods.

**Keywords:** Delay differential equations, periodic points, chaos, horseshoes.

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**Title:** Attracting Fixed Points for Kuramoto-Sivashinsky Equation - a Computer Assisted Proof.

**Abstract:** We present a computer assisted proof of an existence of several attracting fixed points for the Kuramoto-Sivashinsky equation

\[ u_t = (u^2)_x - u_{xx} - \nu u_{xxxx}, \quad u(x, t) = u(x + 2\pi, t), \quad u(x, t) = -u(-x, t), \]

where $\nu > 0$.

The approach is based on the concept of self-consistent a priori bounds introduced in [2]. The method is general and can be applied to other dissipative PDEs, for example Navier-Stokes or Ginzburg-Landau equations. The partial results concerning a rigorous steady-state bifurcation diagram for Kuramoto-Sivashinsky equation will be also mentioned.

**Keywords:** Dissipative PDEs, fixed points, Galerkin projection, computer assisted proof.

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