Adaptive Multi-Level Monte-Carlo method for stochastic variational inequalities

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Monte-Carlo (MC) methods is a well established and widely used approach in statistical simulations. "Sampling" of stochastic data as the crucial part of the method entails the numerical solution of numerous deterministic problems that makes performance the main weakness of the approach. A more advanced approach and namely Multi-Level Monte-Carlo method was introduced by Giles for stochastic ordinary differential equations of Itô type. The MLMC method uses multigrid ideas that allows reducing computational cost considerably.

In this work we focus on a variation of MLMC method for elliptic variational inequalities with stochastic input data. Adaptive Multi-Level Monte-Carlo Finite Element method combines the ideas of Multi-Level Monte-Carlo Finite Element (MLMC-FE) method and a posteriori error estimation for adaptive solution of deterministic spatial problems. Whereas the classical MLMC-FE method is based on a hierarchy of uniform meshes, in the adaptive version of the method we use meshes generated by adaptive mesh refinement of an initially given mesh. Generation of meshes in the method is based on a posteriori error estimation. In contrast to MLMC-FE method, there is no given notion of mesh size anymore and levels are characterized by a hierarchy of FE-error tolerances. Under suitable assumptions on the problem, convergence of adaptive MLMC-FE method is shown and upper bounds for it's computational cost are obtained. We illustrate our theoretical findings by applying the method to a model stochastic elliptic inequality. Our numerical experiments show a reduction in the computational cost of adaptive MLMC-FE method compared to standard MLMC-FE method. In the end we apply the method to a practically relevant stochastic contact problem with an application in biomechnical investigations of human knee implants. Although we focus on elliptic variational inequalities, the method can be used for solution of other problems containing partial derivatives.