Propelinear Codes, Perfect Codes and Partitions

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Abstract

Coding Theory is one of the subjects of interest for the *Combinatorics and Digital Communication group*, among several others (Criptology, Computer Security, Combinatorics, Graph Theory, Data Compression...).

The starting point was the study of connections between completely regular codes and distance regular graphs [6, 7]. From these relations, it appears a new and key concept for our research: The class of propelinear codes. Any propelinear code has a group structure and a linear code is a special case of propelinear code.

An interesting subclass of propelinear codes is the class of binary translation invariant propelinear codes. As can be seen in [8], this class includes linear, $\mathbb{Z}_4$-linear and quaternion codes. Moreover, any translation invariant propelinear code is a combination of codes of these types. If a translation invariant propelinear code has no quaternion part, then it is Abelian. The interest for studying codes of this kind is pointed out in [3].

More recently, we are interested in binary 1-perfect codes and partitions. We have proven (see [1]) that an additive code (in the sense of [2]) is a translation invariant propelinear code (with no quaternion part). Also, any 1-perfect translation invariant propelinear code is an additive code. This equivalence enabled us to give a full characterization of 1-perfect additive codes ([1]).

In [5], it is proven that a binary 1-perfect partition has always a quasigroup structure which is a homomorphic image of a simple algebraic structure defined on the n-cube. Some Steiner triple systems allow to define this algebraic structure and to construct 1-perfect partitions of the n-cube. For length 15, there are 80 nonisomorphic Steiner triple systems (see [11]), 16 of these allow the construction mentioned above.

Following this direction, we considered the problem of finding 1-perfect partitions containing a given binary 1-perfect code ([10]). In [9], we study 1-perfect partitions obtained from binary 1-perfect propelinear codes. We define the classes of uniform and distance-preserving 1-perfect partitions. The main result in [9] is the equivalence between 1-perfect propelinear codes and 1-perfect uniform partitions; and between 1-perfect translation invariant propelinear codes and 1-perfect distance-preserving uniform partitions. The existence of 1-perfect uniform nondistance-preserving partitions, and so the existence of 1-perfect nontranslation invariant propelinear codes, is also stated. We also know that there exist 1-perfect nonuniform distance-preserving partitions, but the only examples we have are the 11 nonequivalent 1-perfect partitions of length 7 ([4]). Hence, the existence of such partitions of the n-cube for $n > 7$ is an open question.

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